A hybrid robust-stochastic operation model for BESS in the day-ahead and automatic frequency restoration markets

Matteo Spiller, Andrea Scrocca, Diego Andreotti, Giuliano Rancilio, Filippo Bovera, Marco Merlo
Department of Energy
Politecnico di Milano
Via Lambruschini 4, Milan 20156, Italy

Abstract— **Battery energy storage systems (BESS) are essential for managing the increased penetration of renewable energy sources (RES). The limited energy capacity of these assets requires tools for optimising the stacking of multiple services. This work proposes a hybrid robust-stochastic mixed-integer linear program (MILP) to effectively schedule the participation of BESS in the day-ahead market (DAM) and the automatic frequency restoration market (aFRR).**

The adopted BESS model uses a piecewise linear interpolation to describe the system's efficiency. A robust formulation tackles the uncertainty in the DAM prices, while a stochastic formulation addresses the uncertainty of the aFRR energy request.

The model determines the bids to submit in the day-ahead electricity markets and the aFRR market.

A case study assesses the model's daily profit. The robust formulation reduces the risk of an incorrect DAM decision by more than 63.66%. The stochastic formulation increases the revenues by 31.94%.

***Keywords*—Arbitrage, BESS, aFRR, electricity markets, services-stacking.**

I. NOMENCLATURE

The main symbols used in this paper are defined below.

A. *Indices*

t	Index of time periods running from 1 to 24
s	Index of aFRR scenarios
j	Index of the j-th element of the BESS look-up table

B. *Constants*

$P_j^{dis,DC}$	i-th DC discharging power in the BESS look-up table
$P_j^{dis,AC}$	i-th AC discharging power in the BESS look-up table
$P_j^{cha,DC}$	i-th DC charging power in the BESS look-up table
$P_j^{cha,AC}$	i-th AC charging power in the BESS look-up table
P^{BESS}	BESS nominal power
E^{BESS}	BESS nominal energy
$\overline{DAM_t^{price}}$	Upper bound of the DAM uncertainty set
$\underline{DAM_t^{price}}$	Lower bound of the DAM uncertainty set
$\pi_{t,s}^{UP}$	aFRR upward capacity price
$\pi_{t,s}^{DOWN}$	aFRR downward capacity price

C. *Variables*

$y_{i,t,s}^{dis}$	Discharge interpolating weight
$y_{i,t,s}^{cha}$	Charge interpolating weight
$z_{t,s}^{dis}$	Discharge binary variable
$z_{t,s}^{cha}$	Charge binary variable
z_t^{idle}	Idle binary variable
$p_t^{dis,DC}$	DC discharge power in p.u.
$p_t^{dis,AC}$	AC discharge power in p.u.
$p_t^{cha,DC}$	DC charge power in p.u.
$p_t^{cha,AC}$	AC charge power in p.u.
SOC_t	State of charge
e_t^{BESS}	BESS energy content
$e_{t,s}^{aFRR\ DOWN}$	Downward reserve requested by the aFRR
$e_{t,s}^{aFRR\ UP}$	Upward reserve requested by the aFRR
$e_t^{DAM\ purch}$	Energy purchased in the DAM
$e_t^{DAM\ sold}$	Energy sold in the DAM

II. INTRODUCTION

The decarbonisation targets set by the European Union are leading to the gradual decommissioning of traditional power plants. The installation of inertia-less renewable energy sources (RES) poses significant challenges to the security of the electric power system [1]. Assets capable of storing energy surplus and rapidly counteracting the aleatority of RES are gaining considerable interest in the field. Flywheels, hydrogen, and flow batteries effectively answer those needs. However, only Battery energy storage systems (BESS) currently represent a market-ready solution [2].

Large-scale battery energy storage systems (BESS) have demonstrated significant potential in electricity markets due to their flexibility, modularity, and rapid response capabilities. These attributes allow BESS to deliver various grid services, such as intertemporal arbitrage, multi-frequency control [3] and reactive power provision [4].

Tools that efficiently stack multiple services while managing the uncertainty of RES and multiple markets are essential for enhancing the economic viability of BESS [5]. Mixed-integer linear programming (MILP) is the commonly used method to optimise BESS scheduling under uncertainty.

979-8-3315-1278-1/25/$31.00 ©2025 IEEE

The authors in [6] proposed hybrid stochastic-robust optimisation in the day-ahead market (DAM) and real-time market (RT). Mancini et al. [7] developed a three-stage stochastic optimisation to optimise the wind-farm BESS power plant bids participating in the day-ahead and automatic frequency restoration reserve markets (aFRR). Various works in the literature focus on bidding strategies for RES-BESS power plants participating in DAM and balancing markets (BM) [8-10]. However, to the authors' knowledge, existing approaches have never investigated hybrid robust-stochastic formulations for optimising the operation of BESS in the DAM and aFRR markets.

This work develops a 24-hour scheduling model to enhance BESS bidding opportunities in the Italian electricity markets. We propose an energy management system that determines the globally optimal bidding strategy for a stand-alone BESS participating in the DAM and aFRR markets. The BESS formulation builds on an updated version of the linearised model proposed in [11]. Robust optimisation accounts for DAM price uncertainty, while stochastic formulation manages the acceptance and the power activation mandated by the aFRR. The tool outputs are optimal DAM bids, and the capacity allows economically efficient participation in the aFRR market.

The structure of this work is as follows. Section III describes the methodology used to develop the optimisation framework. Section IV presents the results of the optimisation models. Section V summarises the outcomes and outlines future works.

III. METHODOLOGY

A. Approach

The developed MILP problem is a multi-objective hybrid robust-stochastic optimisation with a sampling period of 15 minutes. The first component of the objective function minimises the bidding cost in the DAM with a robust formulation. The results of this section are the most efficient bid to place in the DAM accounting price uncertainty. The second part of the objective function minimises the expected value of the aFRR scenarios and determines the upward and downward capacity to submit in the aFRR market.

The following paragraphs describe the methodology used to generate the problem inputs: the DAM uncertainty set and the aFRR scenarios. The consequent sections detail the structure of the MILP: the BESS model, the DAM robust formulation, the aFRR stochastic formulation, and the complete model.

B. DAM and uncertainty set

The Italian spot electricity market consists of three sequential sections: The DAM, the Intra-day market (IM), and the ancillary services market (ASM). The day-ahead market closes at noon on the day before delivery and determines the Italian zonal market prices through a single auction mechanism. Arbitrage leverages the differences between the hourly clearing prices to generate value. Therefore, tools capable of accurately forecasting the prices play a key role in energy arbitrage. In recent years, the volatility of prices surged due to the European gas crisis [12]. This condition raises important challenges in forecasting the prices, driving the need for approaches capable of coping with the risk of an imprecise price estimation.

Robust optimisation considers the worst-case scenarios of the parameters within a predefined uncertainty set to identify a risk-aware solution. This work exploits a polyhedral uncertainty set to describe the DAM. The inputs are the 2024 DAM prices from the Italian nominated energy market operator's (NEMO) website [13]. The adopted approach does not exploit any forecast algorithms as they are outside the scope of this work. The methodology identifies the upper ($\overline{DAM_t^{price}}$) and lower ($\underline{DAM_t^{price}}$) bounds of the uncertainty set by analysing the DAM realisation from the previous seven days. The procedure constructs the uncertainty set using the minimum and the maximum prices observed over these seven days.

The procedure results are two vectors containing 96 values, determining the robust formulation's uncertainty set. Since the clearing process of the DAM defines the hourly prices, each vector has the same value coupled in a set of four.

C. aFRR scenarios

The transmission system operator (TSO) utilises the aFRR market to procure the capacity for system stabilisation. This market is fundamental to maintaining the system frequency at its nominal value and ensuring that interzonal exchanges align with the values set by the DAM and IM. After purchasing the capacity, the TSO imposes a real-time power signal, known as area control error (ACE), on the accepted units to achieve these objectives. The Italian Grid Code requires that the energy exchanged for the aFRR service is compensated at the prices offered by the unit in the ASM. Therefore, to model the aFRR accurately, it is necessary to characterise different parameters: the probability of acceptance, the power activation requested by the ACE, and the remuneration for the units to provide a specific amount of energy.

The data gathered from the Italian NEMO website allowed for a limited analysis of the service's acceptance and remuneration. This condition led to the adoption of deterministic values for these parameters. The acceptance of the offers in the aFRR market has been set at 20 %. Instead, the prices inserted in the optimisation are the average remuneration reported in the data for upward and downward reserve. On the other hand, the power activation of the ACE exploited a stochastic formulation. This optimisation suits the inherent uncertainty of the aFRR market well. Although the power imposed by the ACE follows some deterministic market rules [14], the signal cannot be modelled uniquely. Therefore, scenario-based optimisation allows for an adequate description of ACE's future expectations.

The input data used to generate the ACE scenarios comes from the Italian TSO websites [15]. ACE data refers to 2024. The data has been processed to build two probability density functions (PDF): upward and downward energy activation reserve. These curves are the basis for generating the aFRR energy activation inside the optimisation. . Firstly, since the ACE has a 1-minute time sampling, resampling at 15-minute intervals was necessary to accommodate the optimisation time frame. The 15-minute values represent the sum of all the ACE within that specific quarter-hour. Equation 1 computes the expressions adopted to define upward and downward energy requested by the aFRR for 2024.

[kW]
$$\begin{cases} if\ ACE_t > 50;\ e_t^{UP\ aFRR} = \frac{ACE-50}{50} \\ else\ if\ ACE_t < 50;\ e_t^{DOWN\ aFRR} = \frac{50-ACE}{50} \end{cases} \quad (1)$$

The energies obtained from Equation 1 generate the PDF shown in Figure 1. Lastly, Algorithm 1 processes the PDF and the acceptance rate to develop the scenario of aFRR energy activation. Algorithm 1 describes only the generation of upward activation of aFRR for brevity.

$Algorithm\ 1$
$Input: e_t^{UP\ aFRR}, e_t^{DOWN\ aFRR}$
$for\ t = 1{:}\ N^{scenarios}$
$\quad for\ s = 1{:}\ n^{scenario}$
$\quad\quad if\ remainder\left(\frac{t}{4}\right) = 0$
$\quad\quad\quad acceptance^{upward} = prob\ (20\ \%)$
$\quad\quad e_{t,s}^{aFRR\ UP} = random(PDF^{upward}) * acceptance^{upward}$

Every scenario has its energy activation. The Italian market rule mandates hourly participation in the service to the units accepted. Therefore, every hour, a random extraction evaluates the system's acceptance in the market with a probability of 20%. Lastly, based on the acceptance outcome, the approach samples the PDF and generates an aFRR energy activation. The procedure results are two times the number of scenarios adopted. Each vector describes the energy requested by the upward or downward aFRR service. All the scenarios adopted for the optimisation have the same probability of occurrence w_s.

D. BESS model

In the literature, BESS models use an empirical formulation to describe the behaviour of the asset inside MILP. The systems, modelled as tanks, often adopt a constant charge/discharge process. However, BESS are non-linear technologies whose performance is primarily a function of state of charge (SOC) and power. The authors in [16] considered the impact of the SOC performances negligible. Therefore, the 45-point LUT proposed in [11], which describes efficiency as a function of power and SOC, has been reduced to a four-point power-dependent LUT. This solution allows us to model the performance of the BESS in the function of the sole power. Moreover, it ensures accuracy while reducing computational effort. Table 1 lists the breakpoints adopted to interpolate the power using the special ordered set 2 (SOS2).

Table 1; Breakpoints for the SOS2 efficiency interpolation.

j-Point	$P_j^{dis\ DC}$	$P_j^{dis\ AC}$	$P_j^{cha\ DC}$	$P_j^{cha\ AC}$
1	0.06957	0.05	0.03597	0.05
2	0.09943	0.09	0.08148	0.09
3	0.37401	0.36	0.34653	0.36
4	1.07395	1	• 0.93120	1

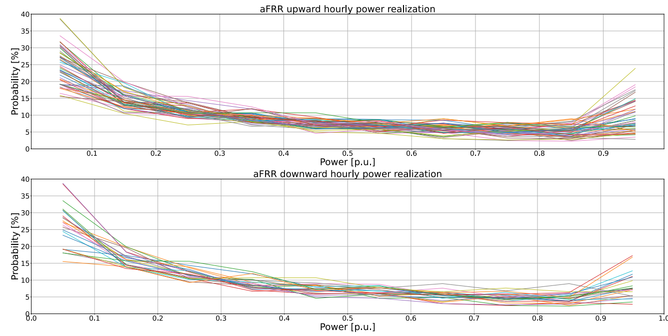

Figure 1; PDF for the energy requested by the aFRR.

The system's efficiency is a by-product of the interpolation described by the set of equations 2.

[Bool] $$\sum_i y_{i,t,s} = z_{t,s}^{dis} \quad 0 \le y_{i,t,s} \le 1\ \ \forall t \quad (2a)$$

[Bool] $$z_{t,s}^{dis} + z_{t,s}^{cha} + z_{t,s}^{idle} \le 1\ \ \forall t \quad (2b)$$

[p.u.] $$p_{t,s}^{dis,DC} = \sum_j P_j^{dis,DC} * y_{i,t,s}^{dis}\ \ \forall t \quad (2c)$$

[p.u] $$p_{t,s}^{dis,AC} = \sum_j P_j^{dis,AC} * y_{i,t,s}{}^{dis}\ \ \forall t \quad (2d)$$

[kW] $$p_{t,s}^{dis\ AC\ kW} = p_{t,s}^{dis\ AC} * P^{BESS} \quad (2e)$$

[kWh] $$e_{t,s}^{BESS} = e_{t-1,s}^{BESS} + \left(p_{t,s}^{cha,DC} - p_{t,s}^{dis,DC}\right) * P^{BESS,nom} * \Delta t \quad (2f)$$

[kWh] $$E^{BESS,nom} * soc_{t,s} = e_{t,s}^{BESS} \quad (2g)$$

[p.u] $$soc_{end\ of\ the\ day,s} = 0.5 \quad (2h)$$

Equation 2a associates the interpolation weight $y_{i,t,s}$ with a specific state of the BESS (charge/discharge). Equation 2b ensures that the BESS can be in only one state at a time: charge, discharge, idle. This solution improved the interpolation's performance compared to the traditional dichotomy charge-discharge. Equations 2c and 2d interpolate the DC and the AC power with the SOS2. Equation 2e converts the per unit power computed by the SOS2 to kilowatts. Equation 2f describes the energy content of the system throughout time. Δt is equal to ¼ to accommodate the 15-minute sampling period of the problem. Expression 2g constrains the energy content with the product between the SOC and the nominal energy. Lastly, equation 2h imposes a SOC equal to 50 % at the end of the day.

E. DAM Robust formulation

The first part of the proposed model uses robust optimisation to tackle the uncertainty of DAM prices. The set of equations 3 detail the objective function and the constraints adopted to implement the robust optimisation problem. This section of the model outputs the optimal offers and bids to be submitted in the DAM. We assume that the system acts as a price taker. Therefore, the bids and offers submitted are always accepted.

$$O.F: min - \sum_t (e_t^{DAM\ sold}) * \frac{\overline{DAM_t^{price}} + \underline{\underline{DAM_t^{price}}}}{2} \sum_t (e_t^{DAM\ purch}) * \frac{\overline{DAM_t^{price}} + \underline{\underline{DAM_t^{price}}}}{2} + z_0 * \Gamma_0 + \sum_t q_t^0 \tag{3a}$$

$$s.t.$$

[kWh] $$z_0 + q_t^0 \geq \left(\frac{\overline{DAM_t^{price}} + \underline{\underline{DAM_t^{price}}}}{2} \right) * y_t^{DA\ SOLD} \tag{3b}$$

$$z_0 + q_t^0 \geq (\overline{DAM_t^{price}} - \underline{DAM_t^{price}}) * y_t^{DA} \tag{3c}$$

$$e_t^{DAM\ sold} - e_t^{DAM\ purch} \leq y_t^{DA} \tag{3d}$$

The model presented applies strong duality theory to linearise the standard robust counterpart of the problem [17]. Equation 3a is the objective function of the optimisation problem. The function minimises the cost of bidding in the DAM while accounting for the uncertainty of the prices. It comprises two parts. The first one minimises the costs. The second one is the worst case of the cost deviation resulting from the possible price deviations. The expression allows the implementation of the polyhedral uncertainty set. Γ_0 adjust the robustness of the formulation against the uncertainty. Γ_0 takes non-negative values within the interval $[0, J^0]$, where $J^0$is a set involving all uncertain coefficients in the objective function. Since the model optimises one-day operation with a sampling period of fifteen minutes J^0 can be chosen within the interval [0,96]. Using strong duality, linear constraints (3b)-(3d) handle the uncertainty of prices affecting the objective function. [18] provides a complete description of the modelling adopted.

F. *aFRR stochastic formulation*

The stochastic model copes with the intrinsic uncertainty in the energy requested by the aFRR service to determine the optimal reserved capacity. The set of equations 4 describes the stochastic formulation implemented following the rules of the Italian electricity market.

$$O.F: \min \sum_s \sum_t w_s * (e_{t,s}^{up\ aFRR} * \pi_{t,s}^{p\ up} - e_{t,s}^{down\ aFRR} * \pi_{t,s}^{p\ down}) \tag{4a}$$

$$s.t.$$

[kWh] $$e_{t,s}^{up\ aFRR} = C_t^{up} * e_{t,s}^{UP\ aFRR} \tag{4b}$$

[kWh] $$e_{t,s}^{down\ aFRR} = C_t^{down} * e_{t,s}^{DOWN\ aFRR} \tag{4c}$$

Equation 4a minimises the expected cost of participating in the aFRR market. The objective function computes the expected value of the aFRR energy activation. Equation 4b and 4c impose the upward and downward energy activation. The aFRR energy activation equals the product of the aFRR capacity selected by the problem and the scenario-specific energy activation.

G. *Hybrid robust-stochastic model and study case*

The robust and stochastic models described in the previous sections operate together to optimise the BESS performances in the Italian electricity market. The following set of equations describes the complete formulation of the model proposed.

$$O.F: \min(3a) + (4a)$$
$$s.t.$$
$$(2a - 2h)$$
$$(3b - 3d)$$
$$(4b - 4c)$$

[kWh] $$p_{t,s}^{dis\ AC\ kW} + e_t^{DAM\ purch} + e_{t,s}^{aFRR\ DOWN} = p_{t,s}^{cha\ AC\ kW} + e_t^{DAM\ sold} + e_{t,s}^{aFRR\ UP} \tag{5}$$

Equation 5 imposes the energy balance of the BESS, ensuring the balance between BESS power and market participation. Table 2 lists the parameters adopted in the study case. The main hypothesis is that the BESS is sited in the NORD Italian bidding zone. The system participates in the Italian DAM and the aFRR markets. We used a PC with an Intel Xeon (3.7 GHz) and 32 GB RAM to solve the model.

Table 2; Optimisation parameters.

Parameters	Value
$E^{BESS,nom}$ [MWh]	2.4
$P^{BESS,nom}$ [MW]	1
Number of aFRR scenario	10
Market zone	NORD
Day investigated	5th May 2024

IV. Results

This section describes the results obtained from the implemented model. First, we focus on the inputs that feed the model. The second section provides details of the outcomes generated by the hybrid robust-stochastic model.

Figure 2 depicts the DAM uncertainty set developed to test the model. The green and blue lines express the uncertainty set's upper and lower bounds. The red line represents the actual realisation of the day under analysis. It is possible to observe that the actual outcome of the DAM is outside of the uncertainty set. However, the methodology specified allows to describe the market trends adequately.

Figure 3 shows an example of the aFRR scenarios generated. The plot represents a limited number of scenarios for the sake of clarity. The positive power is the aFRR upward reserve, while the negative is the downward reserve.

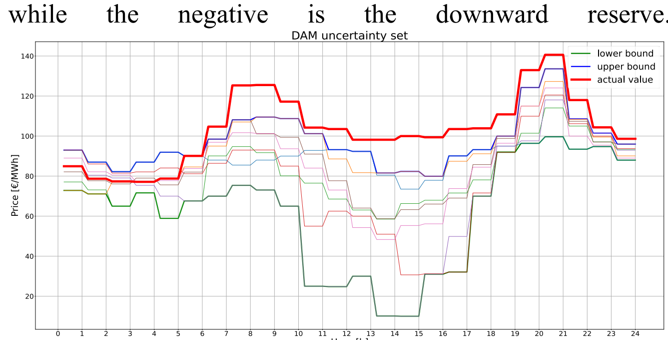

Figure 2; DAM uncertainty set for the robust formulation.

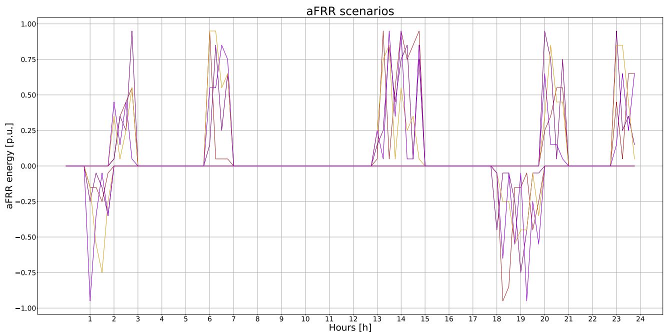

Figure 3; aFRR scenarios generated by the methodology developed.

Figure 4 depicts the outcome of the model obtained with the maximum level of robustness ($\Gamma = 96$). The upper part represents the evolution of the BESS SOC under the different scenarios, accounting for the BESS participation in the DAM and aFRR markets. The figure in the middle shows the optimal bids and the offers to submit in the DAM, considering the previously defined uncertainty set. In yellow and blue, the plot at the bottom represents the capacity to submit to the aFRR market, considering the energy scenarios adopted as an input.

Lastly, we analyse a second simulation with $\Gamma = 0$ to validate the robust formulation of the model. The bids and offers submitted by the deterministic version of the model have a more significant amount of energy. The solution exploits the perfect knowledge of the DAM prices. However, this condition exposes the asset to a wrong price estimation risk. Robust optimisation reduces the energy bought and sold in a specific hour and spreads the offers to reduce the risk of a wrong estimation in one particular hour. The computational effort to solve the model with maximum and zero robustness equals 57 and 63 seconds. Table 3 compares the economic results of the simulation under different levels of robustness and without aFRR where the BESS participates only in the DAM. It can be observed that the robust formulation reduces the revenues from the DAM. However, when computing the profits generated by realising the DAM prices (the red line of Figure 2), the robust formulation protects against uncertainty.

Indeed, the revenue relative error between the optimisation outcome and the actual income is 2.4 % for the robust formulation and 65.99 % for the deterministic model.

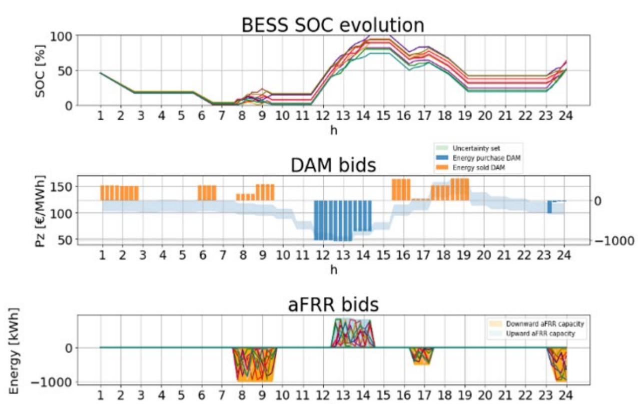

Figure 4; Hybrid robust-stochastic model outputs.

Table 3; Comparison of the economic results with the minimum and maximum robustness value.

Results	Γ=0	Γ=96	Γ=96 no aFRR
DAM revenues	63.11	50.15	75.12
DAM revenues with the price actual realisation	21.71	48.97	64.18
aFRR expected value	50.12	57.48	0
Total expected revenues (DAM+aFRR)	113.23	99.12	75.12

The expected value from the aFRR market is higher with the robust formulation than the deterministic one. Moreover, aFRR increases the daily revenues of the system by 31.94 %. The risk-free formulation of the stochastic approach could justify this outcome. The optimisation relies more on the aFRR market since the outcomes are more reliable when the DAM is computed with maximum robustness. However, future works should also consider risk in the stochastic formulation and investigate the dependency of the expected value with the level of robustness.

V. Conclusions

This work developed a hybrid robust stochastic model for BESS. The model's outcomes are the bids and the offers to submit in the DAM and aFRR markets. The results proved that the robust formulation allowed the proper handling of the uncertainty of the system. Indeed, robustness reduced the risk of economic loss for the BESS in the tested case study. Furthermore, the stochastic formulation allows the stack of multiple services and increases the system's profit.

Future work will expand the analysis to a wider range of study cases. These new cases will enable the evaluation of the dependency between the robust and stochastic parameters. Furthermore, the input creation for both the DAM and aFRR markets could be improved with forecast models. A better forecast algorithm will automatically enhance the optimisation model results, granting revenue enhancement. The MILP model should also include cycle and calendar ageing to constrain the BESS based on the main ageing factors. Lastly, constrained value at risk or similar approaches could be implemented to include risk in the stochastic formulation.

References

[1] PAPAEFTHYMIOU, Georgios; DRAGOON, Ken. Towards 100% renewable energy systems: Uncapping power system flexibility. *Energy Policy*, 2016, 92: 69-82.

[2] BAHLOUL, Mohamed, et al. Analysis of barriers and key enablers toward citizen ESS successful integration. *Journal of Energy Storage*, 2024, 86: 111166.

[3] ZHANG, Mingze, et al. Optimal day-ahead large-scale battery dispatch model for multi-regulation participation considering full timescale uncertainties. *Renewable and Sustainable Energy Reviews*, 2024, 189: 113963.

[4] FARAHANI, Mohammad; SAMIMI, Abouzar; SHATERI, Hossein. Robust bidding strategy of battery energy storage

system (BESS) in joint active and reactive power of day-ahead and real-time markets. *Journal of Energy Storage*, 2023, 59: 106520.
[5] ENGLBERGER, Stefan; JOSSEN, Andreas; HESSE, Holger. Unlocking the potential of battery storage with the dynamic stacking of multiple applications. *Cell reports physical science*, 2020, 1.11.
[6] AKBARI-DIBAVAR, Alireza; ZARE, Kazem; NOJAVAN, Sayyad. A hybrid stochastic-robust optimisation approach for energy storage arbitrage in day-ahead and real-time markets. *Sustainable Cities and Society*, 2019, 49: 101600.
[7] MANCINI, Gianluca, et al. Wind-Battery Pool Optimal Bidding in German Energy and Secondary Control Reserve Markets. In: *2024 20th International Conference on the European Energy Market (EEM)*. IEEE, 2024. p. 1-9.]
[8] TZIOVANI, Lysandros; HADJIDEMETRIOU, Lenos; TIMOTHEOU, Stelios. Optimising the bidding strategy and assessing profitability of over-install renewable plants equipped with battery energy storage systems. *Renewable Energy*, 2024, 234: 121247.
[9] BIAN, Jing, et al. Optimal Bidding Strategy for PV and BESSs in Joint Energy and Frequency Regulation Markets Considering Carbon Reduction Benefits. *Journal of Modern Power Systems and Clean Energy*, 2024.
[10] SCROCCA, Andrea, et al. Techno-economic optimisation of services stacking for a battery participating to electricity spot markets. In: *2024 20th International Conference on the European Energy Market (EEM)*. IEEE, 2024. p. 1-10.
[11] BOVERA, Filippo, et al. Development, validation, and testing of advanced mathematical models for the optimisation of BESS operation. *Sustainable Energy, Grids and Networks*, 2023, 36: 101152.
[12] EMILIOZZI, Simone; FERRIANI, Fabrizio; GAZZANI, Andrea. The European energy crisis and the consequences for the global natural gas market. *The Energy Journal*, 2025, 46.1: 119-145. [13] Bovera, Filippo, et al. "Development, validation, and testing of advanced mathematical models for the optimisation of BESS operation." *Sustainable Energy, Grids and Networks* 36 (2023): 101152.
[13] GME website.
[14] WEIßBACH, Tobias; REMPPIS, Simon; LENS, Hendrik. Impact of current market developments in Europe on deterministic grid frequency deviations and frequency restauration reserve demand. In: *2018 15th International Conference on the European Energy Market (EEM)*. IEEE, 2018. p. 1-6.
[15] Terna Sunset.
[16] VASYLYEV, Andriy; VANNONI, Alberto; SORCE, Alessandro. Optimal dispatch of Li-Ion battery energy storage, reviewing and considering cycling and calendar ageing models. *Applied Thermal Engineering*, 2025, 125597.
[17] BERTSIMAS, Dimitris; SIM, Melvyn. The price of robustness. *Operations research*, 2004, 52.1: 35-53.
[18] RAHIMIYAN, Morteza; BARINGO, Luis. Strategic bidding for a virtual power plant in the day-ahead and real-time markets: A price-taker robust optimisation approach. *IEEE Transactions on Power Systems*, 2015, 31.4: 2676-2687.