Integration of Energy Communities via Multi-level Dynamic Markets

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***Abstract*—Energy Communities (ECs) are groups of prosumers engaging in local energy trading, typically within the shared infrastructure (e.g., behind the same transformer). Local energy markets have emerged as a solution to balance energy demand and production, assuming ECs are price-takers on the main grid. These markets empower prosumers to set local prices and provide scalable pathways for small prosumers to participate in energy systems. As ECs become more prominent, they may transition from price-takers to active market participants, interacting with broader energy markets. This paper investigates interconnected ECs, each operating local energy markets while trading energy through limited transmission infrastructure, and proposes an integrated two-level market optimization framework: local markets set energy prices, while a system-wide market manages transmission capacity and energy exchanges between ECs, maximizing societal welfare. The approach differs from traditional day-ahead markets by optimizing local prices in a time-coupled manner, considering short-term flexibility resources such as batteries and flexible demand. Using a rolling-horizon Model Predictive Control (MPC) scheme, the framework integrates day-ahead and real-time balancing market, reducing planning errors and improving market efficiency. Additionally, we present a Second-Order Optimization Method that rapidly clears prices at both local and system levels, accounting for time coupling. This framework supports dynamic and efficient energy trading, paving the way for ECs to participate in larger energy markets while maintaining operational efficiency.**

***Index Terms*—Local energy market, Battery energy storage, Energy Community, Shared energy storage system.**

I. INTROODUCTION

The energy landscape is undergoing a significant transformation, driven by the increasing integration of renewable resources into power systems worldwide. This shift is largely in response to the challenges posed by the use of fossil resources and concerns over global warming [1]. A key development in this transition is the emergence of Microgrids (MGs) and multi-energy microgrids (MEMGs), which are assemblies of Renewable Energy Sources (RES) such as wind and solar energy, non-RESs like microturbines (MT) and fuel cells (FC), and energy storage systems (ESS). These MGs and MEMGs, despite their source and demand uncertainties, are seen as crucial technologies for accommodating renewable distributed energy resources (DERs) and introducing multiple forms of energy sources into the electricity market [2]. However, the success of this energy transition wave hinges on the acceptance and support of citizens. This is where the concept of EC comes into play [3]. ECs, defined as organizations initiated and managed by civil society actors, aim to educate or facilitate people on efficient energy use, enable the collective procurement of renewable energy or technologies, or provide energy derived from renewable resources for consumption by end-users, participants, or members. These communities present an emergent phenomenon, offering numerous opportunities for citizens to actively participate not only in the community but also in the energy market [4]. In addition to their role as energy consumers, prosumers can take on various roles within the energy market, including deciding on the form and extent to which energy is produced. This active involvement in energy projects can attract funding from residents and investors who feel confident investing due to this local involvement [5]. Furthermore, ECs have the potential to contribute to energy and climate goals, promote collaborative social transformation, and speed up the transition to a low-carbon economy. They can lead local communities to pursue common goals such as energy cost reduction and energy self-sufficiency, play a relevant role in local economic growth, boost smart grid infrastructures, and provide valuable flexibility services to be traded in emerging markets [6].

The concept of ECs is gaining traction as a key element of the green transition. The Clean Energy for All Europeans Package (CEP) released by the European Commission in 2019 has put prosumers at the center of the energy transition, establishing the concept of the EC. In 2020, most EU nations advanced in implementing ECs. ECs are primarily applied to multi-apartment buildings, and countries like Spain, France, and Italy permit the consumers in the EC to produce energy from sustainable sources like rooftop solar, and either sell surplus energy to the grid or share it with neighboring buildings [7]. Greece led EC legislation with Law 4513/2018, establishing a framework for community-led renewable energy initiatives. Forwarded by Law 5037/2023 which is refined EC types and regulations, ensuring clearer governance. [8]. However, some countries have yet to present EC concepts in line with the EU framework. Poland introduced energy clusters in 2015, but it is unclear if this aligns with the CEP requirements [9]. The Netherlands has offered tax relief for cooperative electricity generation since 2013, and Croatia planned to incorporate ECs into its national framework in 2021, and since then, steps have been taken towards their implementation. [10]. Meanwhile, a study commissioned by Nordic Energy Research in 2022 examined the implementation of ECs in the Nordic countries and three other European countries (the Netherlands, Austria, and Germany), explored different models of EC, and assessed conditions that may limit their benefits. [11]. The study aimed to support Nordic authorities in implementing the requirements of Article 16 of the Electricity Market Directive and to facilitate the exchange of views on lessons learned and common experiences in the Nordic and European countries. The study found that few countries have fully transposed the definition of EC into their national legislation, and even fewer use the definition in the general discourse. Most often, they are simply referred to as ECs, encompassing many different models. The study highlighted the importance of clear and coherent definitions in both legal documents and general discourse to lessen uncertainties and hesitation in the initial phase of establishing an EC. Another key finding was that while interest in ECs among the public is increasing, public awareness of the concept remains low [12]. This lack of awareness acts as a barrier to the deployment of ECs. The study recommended introducing clear and coherent definitions of ECs, ensuring accessibility to established ECs, and enabling electricity sharing within the community in an efficient and cost-effective way [13].

In recent years, the local energy market (LEM) in the EC has been viewed as a promising scheme to enhance energy efficiency, promote renewable energy integration, and maximize social welfare at the individual and end-user levels [14]. Several research has been done on designing and evaluating LEM in recent years, and various pricing mechanism approaches have been proposed. For example, the study [15] proposes a transactive energy market pool framework for EC. This framework uses a double auction approach to enable energy trading among various participants. The system includes a periodic double auction pricing mechanism and a distributed load scheduling algorithm for effective demand response management. The study shows that this method significantly improves community self-sufficiency and self-consumption, and also reduces electricity bills for market participants, outperforming previous pricing methods. The proposed pricing mechanism enhances community self-consumption by 10$%–50$% and reduces the community electricity bill by 26$%–40$% under different cases compared to the prosumers to the grid (P2G) market paradigm. A novel auction-based LEM model is developed in [16]. This model is designed to efficiently coordinate the increasing number of prosumers and active participants in the energy system. The authors introduce a new bidding format and investigate various auction-based clearing algorithms. They identify a clearing algorithm that satisfies user preferences, increases local coverage of electricity, and maintains individual rationality and computational tractability. A double auction-based mechanism for a community energy-sharing market was introduced in [17], which comprises distributed prosumers and consumers. The mechanism optimizes battery charging/discharging schedules for community sharing to minimize electricity costs. A two-stage decision-making process is introduced: the first stage involves arbitrage with the utility grid, and the second stage involves real-time market clearing through a non-cooperative game. An adaptive pricing strategy allows agents to update their bids and asks based on historical transaction records and supply and demand forecasting. The case study demonstrates cost savings and increased benefits for agents using the adaptive bidding strategy. A scheduling framework for an EC was developed in [18]. This framework enables energy trading among community members via a local pool, with prices set on a day-ahead basis under the coordination of a Community Manager (CM). Each participant contributes to determining the internal price while managing their scheduling problem amid uncertainty about renewable energy generation and storage. The framework also includes real-time operation and an ex-post settlement of the local market by the CM.

Different from previous studies, which primarily focus on single energy communities or single pricing mechanisms, this paper presents a two-level markets optimization framework for interconnected ECs, made of the local energy markets fixing local energy prices and a system-wide energy market managing the transmission capacity, and fixing the price of global energy exchanges. This market model is based on maximizing societal welfare. Unlike classical day-ahead spot markets, we consider optimizing the local prices in a time-coupled fashion, accounting for the dynamics underlying short-term flexibility resources (batteries, flexible demand) in an economically optimal way. Furthermore, we consider "real-time" local energy markets, whereby the energy prices are updated all the time in a receding-horizon (or MPC) fashion, in order to better manage the uncertainty in the system. This allows combining day-ahead markets and real-time balancing markets information into a single "just-in-time" market mechanism, avoiding the economic losses associated with incorrect planning. The system market is based on the same model and updated jointly with the local markets. Finally, we present an algorithm based on a Second-Order Optimization Method, allowing to clear the prices accounting for time coupling, both at the local and system level with a very fast convergence rate, reducing the computation and communication burden for clearing the markets.

II. Modelling of the Energy Comunity

We denote a group of energy EC K connected in a transmission system, let k index the communities in K where $\mathcal{K} = \{1, 2, \ldots, K\}$. Each EC k consists of a set of prosumers denoted by $\mathcal{N}_k = \{1, 2, \ldots, N_k\}$. In this study, we assume that all prosumers are buildings, meaning they can produce, store, and consume power depending on their available assets, such as photovoltaic (PV) panels, Energy Storage Systems (ESS), and electrical loads (heating systems and base demand). This assumption simplifies the analysis but does not account for other types such as industrial or commercial entities, which may have different energy consumption patterns and flexibility characteristics. Furthermore, the prosumers within each EC can exchange energy locally within their community or trade energy with other ECs to optimize their energy costs and meet their consumption needs. In this study, an MPC scheme is designed to maximize prosumer social welfare over a prediction horizon. The components of this framework, including the prediction model, control objectives, and constraints, are explained in the following subsections.

1) Heating: Heating systems are used in buildings and offer flexible demand capabilities from their inertia. For simplicity, we use a classic two-state building model:

$$T^{\text{in}}_{t+1,k,n} = \alpha_1(T^{st}_{t,k,n} - T^{in}_{t,k,n}) + \beta_1 P^{\text{Heat}}_{t,k,n} \tag{1a}$$

$$T^{\text{st}}_{t+1,k,n} = \alpha_2(T^{in}_{t,k,n} - T^{st}_{t,k,n}) + \alpha_3(T^{out}_{t,k,n} - T^{st}_{t,k,n}) + \beta_2 P^{\text{Heat}}_{t,k,n} \tag{1b}$$

where T^{in}, T^{st}, and T^{out} represent the indoor, structure, and outdoor temperature of the prosumer building. α_1, α_2, α_3, β_1, and β_2 are constant coefficients specific to each building. More complex models can be readily adopted in our algorithms.

The utility function associated to comfort for prosumer n in EC k at time (t) is given as:

$$\Phi^{\text{conf}}_{t,k,n}(T^{in}_{t,k,n}) = -\frac{1}{2}(T^{in}_{t,k,n} - 22)^2 \tag{2}$$

where $\Phi^{\text{conf}}_{t,k,n}(.)$ is the utility function of each prosumer building n at EC k which describes the comfort for the prosumer. T^{in}_t of each building n is constrained by the comfort temperature as mentioned in (2). In this work, we will consider the power scheduling problem associated with each prosumer's heating system n based on the forecasted outdoor temperature T^{out}_t.

2) Energy Storage System (ESS): ESS is of can enhance the operational flexibility of prosumers and potentially provide value-added benefits. However, the extent of these benefits may vary across different regions and market conditions. The utility function of ESS is shown as follows:

$$\Phi^{\text{ESS}}_{t,k,n}(P^{\text{ESS}}_{t,k,n}) = c^{\text{ESS}}_{k,n} \cdot \mid P^{\text{ESS}}_{t,k,n} \mid \tag{3}$$

where $\Phi^{\text{ESS}}_{t,k,n}(.)$ is the operation cost of the ESS for each prosumer building n at EC k which related with power of the ESS at time t where $P^{\text{ESS}}_{t,k,n} < 0$ indicates the discharging, and $P^{\text{ESS}}_{t,k,n} > 0$ signifies charging. $c^{\text{ESS}}_{k,n}$ represents the cost operation of each ESS at EC k. This model applies a simple linear penalty to model ESS operation costs which describes the degradation. However, more accurate models exist but remain debated. Furthermore, this model assumes no cycle loss, meaning that all the energy stored in the ESS can be retrieved without losses. While energy losses due to charging and discharging cycles can be incorporated into the formulation, they are omitted here for simplicity.

The operation cost of the ESS in (3) follows the constraints below:

$$-P^{\text{ESS}_{\min}}_{k,n} \leq P^{\text{ESS}}_{t,k,n} \leq P^{\text{ESS}_{\max}}_{k,n} \tag{4a}$$

$$\text{SOC}^{\text{ESS}_{\min}}_{k,n} \leq \text{SOC}^{\text{ESS}}_{t,k,n} \leq \text{SOC}^{\text{ESS}_{\max}}_{k,n} \tag{4b}$$

$$\text{SOC}^{\text{ESS}}_{t+1,k,n} = \text{SOC}^{\text{ESS}}_{t,k,n} + \left(\frac{P^{\text{ESS}}_{t,k}}{E^{\text{ESS}_{\max}}_{k,n}}\right) \cdot \Delta t \tag{4c}$$

where $P^{\text{ESS}_{\min}}_{k,n}$ and $P^{\text{ESS}_{\max}}_{k,n}$ in (4) are the minimum power and the maximum power of the ESS. $\text{SOC}^{\text{ESS}}_{t,k,n}$ represents the State of Charge of the ESS for each prosumer n in EC k at t. $\text{SOC}^{\text{ESS}_{\min}}_{k,n}$ and $\text{SOC}^{\text{ESS}_{\max}}_{k,n}$ in (5) are the minimum SOC and the maximum SOC of the ESS, respectively. $E^{\text{ESS}_{\max}}_{k,n}$ represents the maximum capacity of the ESS at each prosumer building n. Δt is the sampling time.

The total utility function for each prosumer n in EC k at t can be then written as:

$$\Psi^{\text{Prosumer}}_{t,k,n} = \Phi^{\text{conf}}_{t,k,n}(T^{\text{in}}_{t,k,n}) + \Phi^{\text{ESS}}_{t,k,n}(P^{\text{ESS}}_{t,k,n}) + c_{\text{sys}} \cdot |P^{\text{EC}}_{t,k,n}| \tag{5}$$

where $\Psi_{t,k,n}$ represents the utility function, and c_{sys} is the (fixed) transmission fee within the EC. The absolute value of $P^{\text{ESS}}_{t,k,n}$ is handled via a slack reformulation in the optimization problem. The power balance constraint at t for each prosumer n in EC k is given by:

$$P^{\text{ESS}}_{t,k,n} = P^{\text{PV}}_{t,k,n} + P^{\text{EC}}_{t,k,n} - P^{\text{Demand}}_{t,k,n} - P^{\text{Heat}} \tag{6}$$

where $P^{\text{Demand}}_{t,k,n}$ represents the load demand for prosumer n in EC k at time t, $P^{\text{EC}}_{t,k,n}$ represents the power exported to/from the EC by the prosumer to meet the load demand $P^{\text{PV}}_{t,k,n}$ represents the PV generation from the PV plants and is subjected by the following limitations:

$$0 \leq P^{\text{PV}}_{t,k,n} \leq P^{\text{PV}_{\max}}_{t,k,n} \tag{7}$$

III. Optimization Problem

The optimization framework for interconnected ECs aims to balance local EC energy trading and broader energy exchange between ECs in a way that maximizes societal welfare while respecting system constraints. Each EC consists of a set of prosumers whose total power balance determines the transmission exchange with other ECs. We define the primal variable $\mathbf{P} = \{P^{\text{ESS}}, P^{\text{PV}}, P^{\text{EC}}, P^{\text{Heat}}\}$. The total power generated or consumed by prosumers in an EC, denoted as $\sum_{n=1}^{N_k} P^{\text{EC}}_{t,k,n}$, must match the power exchanged with the broader system, represented as P^{Trans}_k. Each EC has constraints on the amount of power it can transmit or receive, bounded by $-P^{\max}_k$ and $P^{\max}_k$. These limits reflect physical and infrastructure constraints, such as transformers capacity. To ensure consistency across the entire system, the net power exchanged among all ECs must sum to zero, i.e., $\sum_{k=1}^{K} P^{\text{Trans}}_k = 0$. This condition enforces the principle that energy injected into the system equals energy consumed, maintaining overall system balance.

In the centralized framework, the optimization problem can be expressed mathematically as:

$$\min_{\mathbf{P}, P^{\text{Trans}}} \quad \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{n=1}^{N} \Psi_{t,k,n}^{\text{Prosumer}}(\mathbf{P}) \tag{8a}$$
$$\text{s.t.} \quad (1), (4), (6) - (7) \tag{8b}$$
$$\mathbf{S}_{0,k,n} = \mathbf{S}_{\text{initial}} \tag{8c}$$
$$\sum_{n=1}^{N} P_{t,k,n}^{\text{EC}} = P_{t,k}^{\text{Trans}} \tag{8d}$$
$$\sum_{k=1}^{K} P_{t,k}^{\text{Trans}} = 0 \tag{8e}$$
$$- P_k^{\max} \leq P_{t,k}^{\text{Trans}} \leq P_k^{\max} \tag{8f}$$

where $\mathbf{S}_{\text{initial}}$ represents the initial condition of $\mathbf{S}_{\text{initial}}$ = $\{T^{\text{in}}, T^{\text{st}}, \text{SOC}^{\text{ESS}}\}$, P_k^{Trans} is the energy exchanged through the transformer by EC k, and $P_k^{\max}$ is the rated transformer capacity of community k. T is the time horizon which is 24 hours. In this work, we consider $20 \leq T_{t,k,n}^{in} \leq 25$.

If we dualize the problem, we obtain a dual formulation where the constraints are incorporated into the objective function through dual variables, which is described as:

$$\begin{aligned} d(\lambda, \sigma) = \min_{\mathbf{P}, P^{\text{Trans}}} \quad & \sum_{t=1}^{T} \Bigg[\sum_{k=1}^{K} \sum_{n=1}^{N} \Psi_{t,k,n}^{\text{Prosumer}}(\mathbf{P}) \\ & + \sum_{k=1}^{K} \lambda_{t,k} \left(\sum_{n=1}^{N} P_{t,k,n}^{\text{EC}} - P_{t,k}^{\text{Trans}} \right) \\ & + \sigma_t \sum_{k}^{K=1} P_{t,k}^{\text{Trans}} \Bigg] \end{aligned} \tag{9a}$$
$$\text{s.t.} \quad (1), (4), (6) - (7) \tag{9b}$$
$$\mathbf{S}_{0,k,n} = \mathbf{S}_{\text{initial}} \tag{9c}$$
$$- P_k^{\max} \leq P_{t,k}^{\text{Trans}} \leq P_k^{\max} \tag{9d}$$

where $\lambda_{t,k}$ and σ_t describe the dual variable for the constraints (8d) and (8e), respectively. The dual maximization problem can be formulated as:

$$\max_{\lambda, \sigma} d(\lambda, \sigma) \tag{10}$$

The dual maximization problem (10) produces a price λ_k meaning that that constraint (8d) holds total energy import/export within each EC balanced. Similarly, the system price σ arises from enforcing the power balance across ECs in (8e), ensuring that inter-community energy exchanges are settled optimally. Hence, solving the dual problem (10) provides the local and the system prices.

The market clearing process involves decomposing the centralized optimization problem in (9) into independent subproblems for each prosumer n at EC k. Each EC, along with the transformer, independently solves its respective subproblem at the local level. Within this framework, every prosumer in EC determines its local energy trading decisions while ensuring compliance with its specific constraints for each time slot t. The sub-problem for the prosumer n in each EC k can be described as:

$$d_{k,n}(\lambda_k) = \min_{\mathbf{P}_{k,n}} \quad \sum_{t=1}^{T} \left(\Psi_{t,k,n}^{\text{Prosumer}}(\mathbf{P}_{k,n}) + \lambda_{t,k} P_{t,k,n}^{\text{EC}} \right) \tag{11a}$$
$$\text{s.t.} \quad (1), (4), (6) - (7) \tag{11b}$$
$$\mathbf{S}_{0,k,n} = \mathbf{S}_{\text{initial}} \tag{11c}$$

where $\mathbf{P}_{k,n}$ is the primal variables for each prosumer n in EC k. The sub-system problem for each transformer in EC k is an LP given as:

$$d_k^{\text{Trans}}(\lambda_k, \sigma) = \min_{P_k^{\text{Trans}}} \quad \sum_{t=1}^{T} \left(-\lambda_{t,k}.P_{t,k}^{\text{Trans}} + \sigma_t.P_{t,k}^{\text{Trans}} \right) \tag{12a}$$
$$\text{s.t.} \quad - P_k^{\max} \leq P_{t,k}^{\text{Trans}} \leq P_k^{\max} \tag{12b}$$

The dual function in (9) is obtained by summing the independent dual sub-problems at the prosumer and transformer levels. Each prosumer n in an EC k solves a local problem $d_{k,n}(\lambda_k)$, while each transformer optimizes its energy exchange through $d_k^{\text{Trans}}(\lambda_k, \sigma)$, Hence, the overall dual function is expressed as:

$$d(\lambda, \sigma) = \sum_{k=1}^{K} \sum_{n=1}^{N} d_{k,n}(\lambda_k) + \sum_{k=1}^{K} d_k^{\text{Trans}}(\lambda_k, \sigma) \tag{13}$$

where the EC dual functions defined as:

$$d_k(\lambda_k, \sigma) = \sum_{n=1}^{N} d_{k,n}(\lambda_k) + d_k^{\text{Trans}}(\lambda_k, \sigma) \tag{14}$$

The dual maximization problem (9) then reads as:

$$\begin{aligned} \max_{\lambda, \sigma} d(\lambda, \sigma) = \max_{\lambda, \sigma} \sum_{k=1}^{K} d_k(\lambda_k, \sigma) = \\ \max_{\sigma} \sum_{k=1}^{K} \max_{\lambda_k} d_k(\sigma, \lambda_k) \end{aligned} \tag{15}$$

Problem (15) shows that the dual maximization problem can be treated in a hierarchical fashion, where the ECs optimize their local process (λ_k) for given system prices σ, such that the system prices σ can be optimized as a function of the local markets. Thus, EC solves for its optimal local price $\lambda_k^\star$for a given system price σ:

$$\lambda_k^\star(\sigma) = \arg\max_{\lambda_k} d_k(\lambda_k, \sigma). \tag{16}$$

And, the system operator then update σ based on the optimal $\lambda_k^\star(\sigma)$:

$$\sigma^\star = \arg\max_{\sigma} \sum_{k=1}^{K} d_k(\lambda_k^\star(\sigma), \sigma). \tag{17}$$

To reduce computational complexity, we approximate this process by updating λ_k and σ sequentially in each iteration instead of solving each $\lambda_k^\star(\sigma)$ to full convergence before updating σ. While convergence is not theoretically guaranteed, numerical experiments show that this approach yields stable and consistent results.

The next section details the sequential update of the λ_k and σ and the coordination decisions across prosumers, ECs, and transformers to solve the problem in (15) using the Second-Order Optimization Method.

Algorithm 1 Clearing Market Pseudocode Algorithm.

```
Initial guess for σ, and λ.
while ‖ Δσ ‖ and ‖ Δλ_t ‖ < Tol do
  for each EC k in K do
    for prosumer n in N do
      Each prosumer n solves the local problem in (11) for ∇_{λ_k} d_{k,n}(λ_k).
      Each prosumer n computes its sensitivity ∇²_{λ_k} d_{k,n}(λ_k).
      Each prosumer n sends ∇_{λ_k} d_{k,n}(λ_k) and ∇²_{λ_k} d_{k,n}(λ_k) to ECC.
    end for
    Each ECC k solves (12) for ∇d_k^Trans(λ_k, σ)
    Each ECC k calculates the sensitivity ∇²_{λ_k} d_k^Trans(λ_k, σ), and ∇²_σ d_k^Trans(λ,σ).
    Collect and sum:
      Σ_n ∇_{λ_k} d_{k,n}(λ_k, σ) + ∇d_k^Trans(λ_k, σ).
    Collect and sum:
      Σ_n ∇²_{λ_k} d_{k,n}(λ_k, σ) + ∇²d_k^Trans(λ_k, σ).
    Update the local price λ_k for each EC k using (18).
    Send ∇_σ d_k^Trans(λ_k^⋆, σ) and ∇²_σ d_k^Trans(λ_k^⋆, σ) to System Operator.
  end for
  System operator collects and sums:
    Σ_k ∇_σ d_k^Trans(λ_k^⋆, σ).
  System operator collects and sums:
    Σ_k ∇²_σ d_k^Trans(λ_k^⋆, σ).
  Update the system price using (21).
end while
```

IV. Clearing for the Multi-level Dynamic Markets

The overall process is implemented as an iterative clearing mechanism for the multi-level dynamic market which is illustrated in Pseudocode Algorithm 1. Initially, each prosumer forecasts their energy production and consumption, accounting for local resources from PV, ESS, and flexible loads. Based on this forecast, they establish a preliminary schedule by solving the optimization problem locally to maximize social welfare as defined by (11).

Following (15). In the inner loop, each prosumer determines the primal solution of $P^{\mathrm{EC}^\star}$, calculates the corresponding sensitivity $\frac{\partial P^{\mathrm{EC}^\star}}{\partial \lambda}$, and sends both values to the EC coordinator (ECC) to update the local energy price λ for each EC.

Each EC solves the local optimization problem described in Equation (12), determining the primal solution of $P^\star_{\mathrm{trans}}$ and calculating the corresponding sensitivities $\frac{\partial P^\star_{\mathrm{trans}}}{\partial \lambda}$ and $\frac{\partial P^\star_{\mathrm{trans}}}{\partial \sigma}$. The ECC updates the local energy price λ according to the specified update equation.

$$\lambda_k^{\mathrm{iter}+1} = \lambda_k^{\mathrm{iter}} - \alpha_1 . \left(\nabla^2_{\lambda_k} d_k(\lambda_k, \sigma)\right)^{-1} . \left(\nabla_{\lambda_k} d_k(\lambda_k, \sigma)\right) \tag{18}$$

where $\nabla^2_\lambda d(\lambda_k)$ is calculated by summing the sensitivities of all prosumers and ECC in each EC k (ref [13]). α_1 represents the step size. The updated λ values are then broadcast to all prosumers within the EC, ensuring alignment in subsequent iterations. Additionally, the ECC broadcasts $P^\star_{\mathrm{trans}}$ and $\frac{\partial P^\star_{\mathrm{trans}}}{\partial \sigma}$ to the system operator to facilitate the system-level price update.

In the outer loop, the system operator receives the optimal power exchanges and corresponding sensitivity respecting the system price information from each ECC. It is useful to observe from (15) that the local prices $\lambda_k^\star$ implicitly depend on the system price σ, leading to the gradient expression:

$$\nabla_\sigma \sum_{k=1}^{K} \max_{\lambda_k} d_k(\lambda_k, \sigma) = \sum_{k=1}^{K} \nabla_\sigma d_k(\lambda_k^\star, \sigma) \tag{19}$$

In the outer loop, the system operator receives the optimal power exchanges and corresponding sensitivity respecting the system price information from each ECC. From optimization theory, it follows that the local prices $\lambda_k^\star$ are implicitly functions of σ, since they are defined as the maximizers of $d_k(\lambda_k, \sigma)$. As a result, the following second-order derivative expression holds:

$$\begin{aligned}\nabla^2_\sigma \sum_{k=1}^{K} \max_{\lambda_k} d_k(\lambda_k, \sigma) = \sum_{k=1}^{K} \big(&\nabla^2_\sigma d_k(\lambda_k^\star, \sigma) \\ &+ \nabla_\sigma \lambda_k^\star \cdot \nabla^2_{\lambda_k, \sigma} d_k(\lambda_k^\star, \sigma)\big)\end{aligned} \tag{20}$$

It is important to note the second term in (20) is equal to zero in this study because $d_k(\lambda_k, \sigma)$ exhibits a linear relationship between λ_k and σ. As a result, the mixed partial derivative is identically zero. Furthermore, the dual function d^{Trans} is piecewise linear because the underlying optimization problem involves linear constraints and objective functions, and since σ and λ enter linearly in the cost function. Such that its Hessian is zero. In the following, we circumvent that issue by using primal-dual interior point methods to solve the optimization problems and we leverage on the barrier parameter to smooth the dual functions and recover definiteness in the Hessians. Furthermore, the regularization term of the Hessians, such as a weighted identity matrix, could be introduced to improve numerical stability. The system operator updates the system-wide price according to (21).

$$\begin{aligned}\sigma^{\mathrm{iter}+1} = \sigma^{\mathrm{iter}} - \alpha_2 . &\left(\sum_{k=1}^{K} \nabla^2_\sigma d_k(\lambda_k^\star, \sigma)\right)^{-1} \\ &. \left(\sum_{k=1}^{K} \nabla_\sigma d_k(\lambda_k^\star, \sigma)\right)\end{aligned} \tag{21}$$

where the gradient and the approximation of the Hessian for the system-level dual function are defined from (19) and (20) as:

$$\sum_{k=1}^{K} \nabla_\sigma d_k(\lambda_k^\star, \sigma) = \sum_{k=1}^{K} \nabla_\sigma d_k^{\mathrm{Trans}}(\lambda_k^\star, \sigma) \tag{22}$$

$$\sum_{k=1}^{K} \nabla^2_\sigma d_k(\lambda_k^\star, \sigma) = \sum_{k=1}^{K} \nabla^2_\sigma d_k^{\mathrm{Trans}}(\lambda_k^\star, \sigma) \tag{23}$$

where α_2 represents the step size. Integer iter is the iteration number. After updating the system price, the system operator

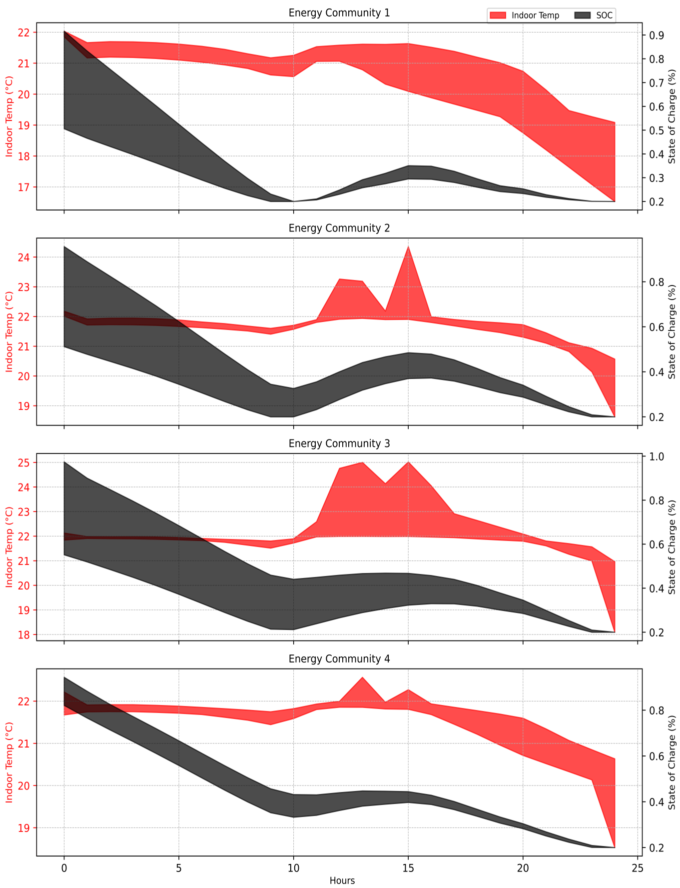

Fig. 1: Optimized Indoor Temperature and SOC ranges for each EC.

broadcasts the revised price σ to each EC, ensuring that the updated information is seamlessly integrated into the local optimization processes. The iterative process reaches its conclusion upon the fulfillment of convergence criteria.

V. Illustrative Example

To test the efficacy of the proposed multilevel dynamic markets and the pricing strategy, we conducted a case study of four connected ECs, each consisting of five prosumers equipped with PV systems, ESS, and heating systems. The DERs have a unity power factor and their nominal capacities are randomly selected, which are between 0 and 10 kW for the PV, and between 0kW/10kWh for the ESS. We set the maximum and the minimum power dispatch of the ESS for each prosumer at 10 and -10 kWh, respectively.

Fig. 1 illustrates the maximum and minimum values of indoor temperature (red) and SOC (black) which are obtained from the optimization results across multiple prosumers within each EC during the 24-h day of operation.

As is observed in Fig. 1. During the 00:00–10:00 period, the SOC is almost always decreasing since there is no PV generation available during that time. The ESS strives to maintain the indoor temperature within the prosumers' comfort range while ensuring that none of the operational limit constraints are violated. From 10:00–17:00, we observe an increase in the SOC due to the availability of PV generation, which can be used to charge the ESS. Additionally, during this period, there is a slight increase in the indoor temperature for some prosumers, which can be attributed to the dynamic thermal response of the prosumer's environment as solar energy becomes more abundant. From 17:00–23:00, the SOC decreases again because the ESS is being utilized to meet the load demand and to maintain indoor temperature comfort for the prosumers. This period demonstrates the discharge cycle of the ESS as it supports evening consumption when PV generation is no longer available.

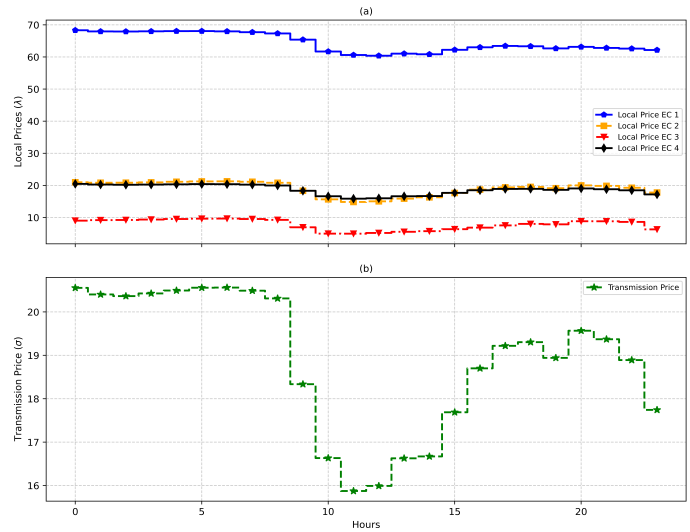

Fig. 2: Market clearing prices for each EC (a) and system (b).

It is worth noting that the optimization exhibits a behavior where SOC and temperature reserves are fully utilized towards the end of the 24-hour horizon, as the model "spends" these resources, assuming the horizon is final. This reflects the lack of consideration for the long-term value of these states beyond the finite MPC horizon.

The market clearing prices during the daily operation are depicted in Fig. 2. Fig. 2 (a) shows the local prices λ for each EC over 24 hours. As observed, EC1 has the highest local price consistently throughout the day, indicating higher energy demand or limited local generation. In contrast, EC2 has the lowest local price, reflecting better balancing of supply and demand within the community or more effective utilization of DERs. Fig. 2 (b) highlights σ across the entire system. The transmission price exhibits a stepped behavior with noticeable increases during peak demand hours, particularly between 16:00 and 20:00, aligning with the evening load peaks. Conversely, the transmission price decreases during the early morning hours (00:00–06:00) when overall system demand is lower. The observed trends reflect the dynamic interactions between local energy generation, storage utilization, and transmission costs. The system's pricing strategy encourages local energy generation and consumption during peak solar hours while reflecting the increased reliance on transmission networks during higher demand periods.

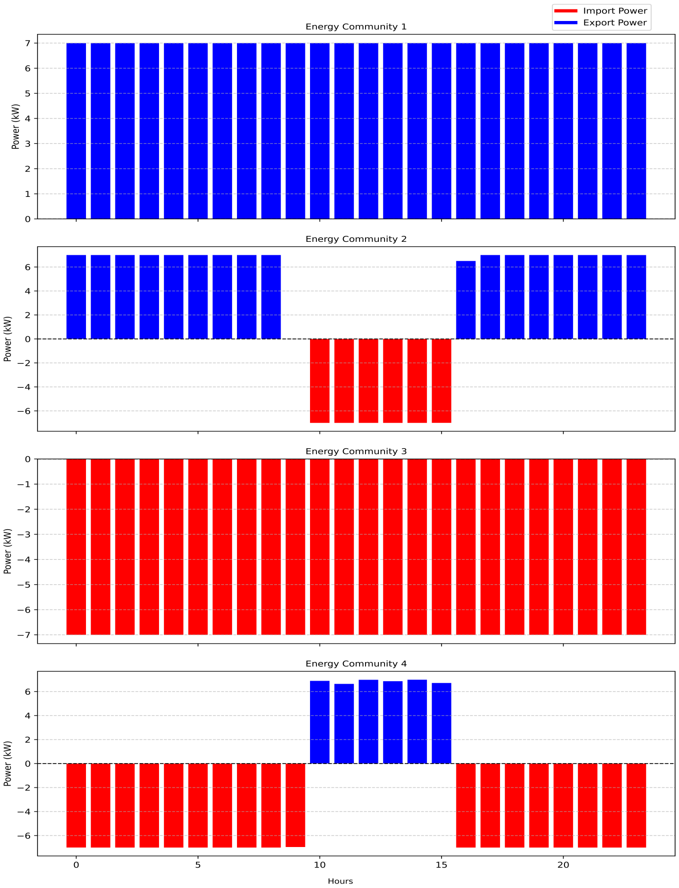

Fig. 3: Volume of the power exchange for each EC.

Fig. 3 illustrates the volume of power exchange between the ECs over 24 hours. The exchange is presented as either imported power or exported power for each community. The results confirm that the power balance across all energy communities is successfully maintained. The total exported power equals the total imported power, demonstrating effective coordination and optimal exchange within the system.

VI. Conclusion

This paper presents a novel two-level market optimization framework for interconnected ECs, enabling efficient local energy trading and system-wide energy exchange. The framework integrates local energy markets with a system-wide market, maximizing societal welfare while accounting for short-term flexibility resources such as ESS and flexible demand such as heating. By employing a rolling-horizon MPC scheme, the approach combines day-ahead and real-time balancing markets, reducing planning errors and improving market efficiency. A Second-Order Optimization Method is introduced to clear prices at both local and system levels. The case study demonstrates the framework's effectiveness in balancing energy demand and production, optimizing energy storage utilization, and maintaining prosumer comfort. The optimization depletes SOC and temperature reserves near the horizon's end, as the MPC framework neglects their value beyond the optimization period.

Future work should incorporate terminal costs for SOC and temperature to reflect their long-term importance. Additionally, improvements in the algorithmic structure are needed to enhance the computational efficiency and robustness, this includes refining the second-order update for the cross dual variables. Moreover,we will explore the integration of line topology in interconnected ECs and develop optimal power flow strategies to enhance energy transaction efficiency and system resilience further.

References

[1] Y. Zahraoui, I. Alhamrouni, S. Mekhilef, M. R. B. Khan, M. Seyedmahmoudian, A. Stojcevski, and B. Horan, "Energy Management System in Microgrids : A Comprehensive Review," *Sustainability*, pp. 1–33, 2021.

[2] B. Han, Y. Zahraoui, M. Mubin, S. Mekhilef, M. Seyedmahmoudian, and A. Stojcevski, "Optimal Strategy for Comfort-Based Home Energy Management System Considering Impact of Battery Degradation Cost Model," *Mathematics*, vol. 11, no. 6, 2023.

[3] Y. Zahraoui, T. Korõtko, A. Rosin, and H. Agabus, "Market Mechanisms and Trading in Microgrid Local Electricity Markets: A Comprehensive Review," *Energies*, vol. 16, no. 5, pp. 1–52, 2023.

[4] Y. Zahraoui, T. Korõtko, A. Rosin, and H. Agabus, "A Competitive Framework for The Participation Of Multi-Microgrids in The Community Energy Trading Market: A Case Study.," *IEEE Access*, vol. 2169-3536, no. 11, p. 1, 2024.

[5] B. Han, Y. Zahraoui, M. Mubin, S. Mekhilef, T. Korõtko, and O. Alshammari, "Distributed optimal storage strategy in the ADMM-based peer-to-peer energy trading considering degradation cost," *Journal of Energy Storage*, vol. 96, no. 96, p. 112651, 2024.

[6] B. Han, Y. Zahraoui, M. Mubin, S. Mekhilef, M. Seyedmahmoudian, and A. Stojcevski, "Home Energy Management Systems : A Review of the Concept , Architecture , and Scheduling Strategies," *IEEE Access*, vol. 11, no. February, pp. 19999–20025, 2023.

[7] IEA, "Italy's renewable energy community initiative," 2023.

[8] "Greece EC legislation," 2023.

[9] D. Frieden, A. Tuerk, J. Roberts, and S. Herbemont, "Collective self-consumption and energy communities : Overview of emerging regulatory approaches in Europe," no. June, 2019.

[10] "ENERGY COMMUNITIES ' CHALLENGES AT THE Policy briefings SCCALE 20 30 50,"

[11] Technopolis, "Energy Communities Nordic Energy resource," vol. 11, no. 1, pp. 1–14, 2019.

[12] Y. Zahraoui, S. Gros, and I. Oleinikova, "Day Ahead Schedule of Multi-Energy Communities Participating in The Local Market Considering Community Energy Storage System,"

[13] S. Gros, D. Reinhardt, J. Rajasekharan, P. Crespo, and del, "Dynamic Spot Markets for Flexible Demand and Energy Communities Via Distributed Optimization,"

[14] Y. Wu, G. Pan, Z. Wu, W. Gu, Z. Xu, B. Yuan, S. Zheng, and Q. Sun, "Decentralized energy market integrating carbon allowance trade and uncertainty balance in energy communities," *Sustainable Energy, Grids and Networks*, vol. 39, no. May, p. 101453, 2024.

[15] H. R. Bokkisam, R. M. Acharya, and M. P. Selvan, "Framework of transactive energy market pool for community energy trading and demand response management using an auction-theoretic approach," *International Journal of Electrical Power and Energy Systems*, vol. 137, no. October 2021, p. 107719, 2022.

[16] M. Zade, S. D. Lumpp, P. Tzscheutschler, and U. Wagner, "Satisfying user preferences in community-based local energy markets — Auction-based clearing approaches," *Applied Energy*, vol. 306, no. PA, p. 118004, 2022.

[17] L. He, G. S. Member, J. Zhang, and S. Member, "A Community Sharing Market With PV and Energy Storage : An Adaptive Bidding-Based Double-Side Auction Mechanism," *IEEE Transactions on Smart Grid*, vol. 12, no. 3, pp. 2450–2461, 2021.

[18] J. L. Crespo-Vazquez, T. Alskaif, A. M. Gonzalez-Rueda, and M. Gibescu, "A Community-Based Energy Market Design Using Decentralized Decision-Making under Uncertainty," *IEEE Transactions on Smart Grid*, vol. 12, no. 2, pp. 1782–1793, 2021.